Physics III
ISI B.Math
Back Paper Exam : Jamuary 4, 2011

Total Marks: 100
Answer all questions.

1.a) Write down the full set of Maxwell's equations in differential form. What is the significance of the asymmetry between the equations involving the divergence of the electric and magnetic fields?(6)
b) Derive the continuity equation, which implies the conservation of charge, from Maxwell's equations. (6)
c) Why is Ampere's law :$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ inconsistent in the general context of electrodynamics? (2)
d)Show that Maxwell's equations imply the following

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

where V is an arbitrary scalar function and $\mathbf{A}$ is an arbitrary vector field. (6)

2. A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance l apart (fig 2). A resistor R is connected across the rails and a uniform magnetic field $\mathbf{B}$, pointing into the page, fills the entire region.
(a) If the bar moves to the right at speed v, what is the current in the resistor? In what direction does it flow? (5)
(b) What is the magnetic force on the bar? In what direction? (4)
(c) If the bar starts out with a speed v_0 at the time $t = 0$, and is left to slide, what is the speed at a later time t?(5)
(d) The initial kinetic energy of the bar was, of course $\frac{1}{2}mv_0^2$. Check that the energy delivered to the resitor is exactly $\frac{1}{2}mv_0^2$.(6)

3. Consider the following electric and magnetic fields whose real parts represent monochromatic plane waves propagating with velocity $c = \omega/k$.

$$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E_0}} e^{i(kz-\omega t)}$$

$$\tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_{\mathbf{0}} e^{i(kz-\omega t)}$$

(i) If the waves satisfy Maxwell's equations, find the relationship between the real amplitudes E_0 and B_0.(6)
(ii) If the waves satisfy Maxwell's equations, show that $\mathbf{B}$ and $\mathbf{E}$ are in phase and mutually perpendicular(6)
(iii) Find the Poynting vector for the above waves (5)
(iv) Write down the modified form of the waves if they were propagating in the x direction.(3)

4.a) A hollow spherical shell carries charge density

$$\rho = \frac{k}{r^2}$$

in the region $a \leq r \leq b$ (fig 4). Find the electric field in the three regions (i)$r < a$,(ii) $a < r < b$, (iii) $r > b$. Plot $|\mathbf{E}|$ as a function of r. (10)
b) A point charge q is held a distance d above an infinite grounded conducting plane.
i) Find the force of attraction between the charge and the plane (5).
ii)Find the energy of the configuration (5)

5.(a) A beam of electrons with charge q and mass m is passed through crossed uniform electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$(mutually perpendicular,and both of them perpendicular to the beam). The electric field is then adjusted until the beam does not get deflected at all. What was the speed of the particles in terms of E and B?(5)
(b) the electric field is then turned off and the radius of curvature R of the beam is measured, as deflected by the magnetic field alone. What is the charge-to-mass ratio $\frac{q}{m}$ of the particles in terms of E, B, and R ?(5)
(c) Calculate the work done by the electrons in the context of part (b),i.e, when the electric field is turned off. (2)
(d)A electric dipole consists of two equal and opposite charges $\pm q$ separated by a distance d. Find the approximate potential $V(r,\theta)$ in the regime $r \gg d$. Consider the dipole to lie along the z axis, with the charge $+q$ at $(\frac{d}{2}, 0)$ and $-q$ at $(-\frac{d}{2}, \pi)$. θ is the angle between the radius vector of any point and the $+z$ axis. From the potential find the electric field of the dipole in the form $\mathbf{E} = E_r\hat{\mathbf{r}} + E_\theta\hat{\theta}$(8)

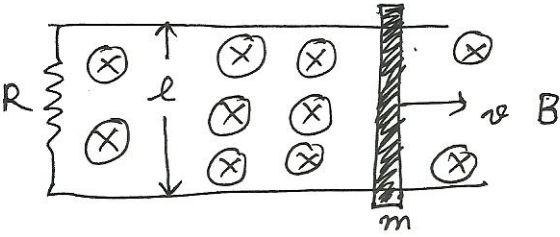

Fig 2.

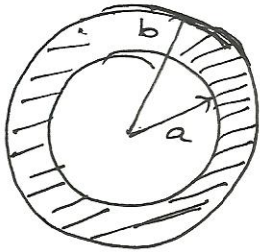

fig. 4a

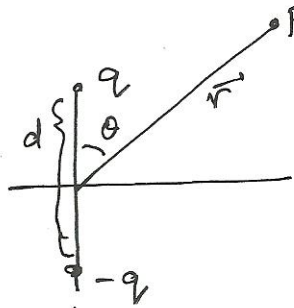

Fig 5d